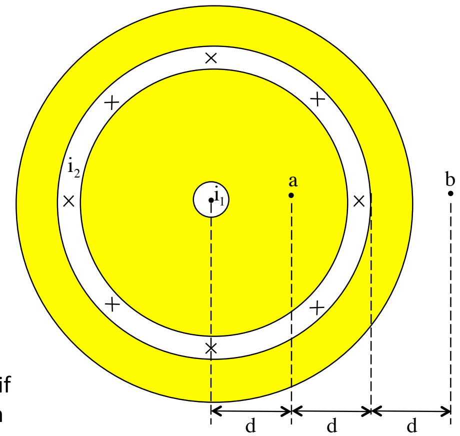
Problem 30.29

What fun!

Think back for a second. Gauss's Law essentially said that if you can determine the electric flux through a closed surface, that flux will be proportional to the amount of charge found *inside* the surface.



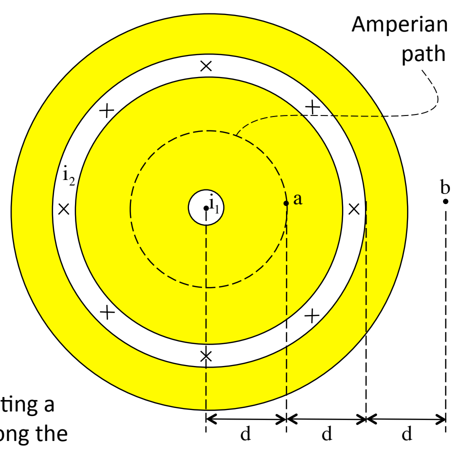
Ampere's Law makes a similarly useful observation. It notices that if you can determine what is called in the world of mathematics "the circulation of the magnetic field" around a close path, that quantity will be proportional to the amount of current that passes *through* the face of the path.

To execute Ampere's Law:

1.) Identify an imaginary path that has "good symmetry" and that passes through the point at which you want to know B. (This path is called an "Amperian path.")

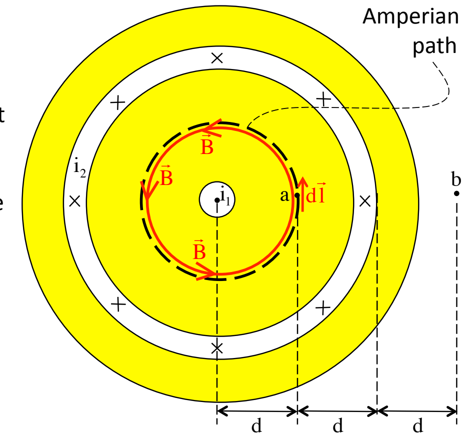
2.) Write out Ampere's Law:

$$\oint \vec{B} \bullet d\vec{l} = \mu_o i_{thruFace}$$



Note that the dot product is generating a product of "the component of $\vec{B}$ along the line of $d\vec{l}$" times that differential path length. Summing that quantity around a entire path gives "the circulation of B around the path." That is the quantity that is proportional to the current passing *through the face of the path*, with μ_o being the proportionality constant that gives us our equality.

Define $d\vec{l}$ as shown on the sketch. Note that $\vec{B}$ is in the same direction as $d\vec{l}$ (hence, the angle between $\vec{B}$ and $d\vec{l}$ is zero and the cosine in the dot product is one). $\vec{B}$'s magnitude is the same everywhere along the path (hence it can be pulled out of the integral because it's a constant). With all of that:



$$\oint \vec{B} \bullet d\vec{l} = \mu_o i_{thruFace}$$

$$\Rightarrow \quad \oint B_{at\,"a"}\, dl \cos 0^\circ = \mu_o i_1$$

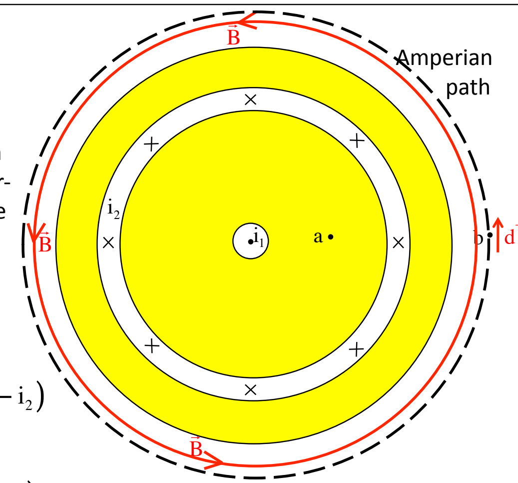
$$\Rightarrow \quad B_{at\,"a"} \oint dl = \mu_o i_1$$

$$\Rightarrow \quad B_{at\,"a"} (2\pi d) = \mu_o i_1$$

$$\Rightarrow \quad B_{at\,"a"} = \frac{\mu_o i_1}{2\pi d} = \frac{(4\pi x10^{-7}\ T \bullet m / A)(1\ A)}{2\pi(.001\ m)}$$

$$= 2x10^{-4}\ T$$

b.) With Gauss's Law, particular geometries always had similar evaluations. Likewise, the only difference between this problem and Part "a" is the amount of current that passes through the face (current into the page is defined as negative) and the distance to the point of interest. So:



$$\oint \vec{B} \bullet d\vec{l} = \mu_o i_{thruFace}$$

$$\Rightarrow \quad \oint B_{at\,"b"}\, dl \cos 0^\circ = \mu_o (i_1 - i_2)$$

$$\Rightarrow \quad B_{at\,"b"} \oint dl = \mu_o (i_1 - i_2)$$

$$\Rightarrow \quad B_{at\,"b"} (2\pi(3d)) = \mu_o (i_1 - i_2)$$

$$\Rightarrow \quad B_{at\,"b"} = \frac{\mu_o (i_1 - i_2)}{6\pi d} = \frac{(4\pi x10^{-7}\ T \bullet m / A)[(1\ A) - (3\ A)]}{6\pi(.001\ m)}$$

$$= -1.33x10^{-4}\ T$$

Interesting side point. The direction of $d\vec{l}$ is usually aligned with the assumed direction of $\vec{B}$ so that dot product comes out to be positive. If the currents through the path's face sum to a negative number, though, you will end up with a calculated value for the magnitude of $\vec{B}$ being *negative*. As magnitudes are always positive, what does the negative sign mean? It means you have assumed the wrong direction for $\vec{B}$. In this case, we assumed that direction was "up" at Point "b." In fact, it is "down."